Notes on Black Hole Information Problem

Aayush Verma

E-mail: aayushverma6380@gmail.com

Abstract: These notes are discussions on the black hole information problem starting from Bekenstein. Among various things, we comment on information problem and some of the proposed solutions. We also discuss the Hayden-Preskill protocol of extracting information out of the black hole.

Contents

1 Introduction

Black Holes are promising physics, and we will dedicate these notes to black holes and their connection with quantum mechanics, thermodynamics, and the famous information problem. A black hole (BH) is where we can observe both general relativity and quantum effects, which is rare in physics. In fact, black holes have now merged with various quantum gravity as well. For classical reviews of black holes, see Ref. [1, 2]. Other interesting reviews are Ref. [3, 4]. The main motivation for studying black holes with quantum effects is primarily because of the inclusion of quantum gravity. In most of these settings, physicists often play with semiclassical models to understand parts of quantum gravity in the region. Sometimes it is not even necessary to study the interior, which is too complex to understand.

For this reason, it is also one of the big questions in quantum gravity [5]. One learns a great amount deal of information theory[1] (and computing) while doing black holes information problem[2]. This has opened a new information dimension to the problem, for instance, works by Hayden, Preskill, Aaronson, and Susskind [8–10].

While it is too difficult to trace the history of black holes and the information problem of black holes, one can understand the motivation for studying black holes. We expect that black holes are the keys to quantum gravity, and a theory that can describe black holes with proper consistency can be called the most precise quantum gravity model.

[1]For a discussion on information theory see [6, 7].
[2]Or paradox, we will use 'problem' rather than a paradox.

2 Setup

The information paradox was initiated by Hawking's works [11, 12]. But before discussing what actually this paradox is, we need to set up the black hole and its thermodynamic properties. We would expect from the second law of thermodynamics that if a cup of tea is tossed into a black hole, then the entropy of the black hole should increase. However, what happens to the entropy decrease of the outside universe? What is the quantity which increases throughout the tossing without violating entropy decrease? The answer was found in the area of the event horizon. One writes the entropy of the black hole as Bekenstein-Hawking entropy[3]

$$S_{BH} = \frac{A}{4G_N}, \tag{2.1}$$

where A is the area of the event horizon, and it is measured in Plank units $l_p^2 = \hbar G_N$. One can, for obvious reasons, which we will discuss later, suspect that there are many degrees of freedom for a black hole. For comparison, the black hole of the size of the proton has an entropy of $S \approx 10^{40}$. We will come back to the thermodynamics discussion of a black hole in Sec. 3.

We now see a coordinates system in which we write the metric for black holes. The famous one is Schwarzchild coordinates, in which we write[4]

$$ds^2 = -\left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{1}{\frac{2MG}{r}}\right) dr^2 + r^2 d\Omega^2. \tag{2.2}$$

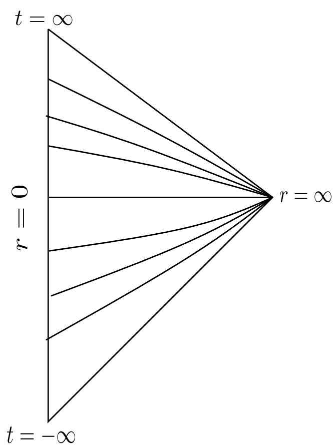

Figure 1. Penrose diagram for Minkowski's space.

The other important coordinate is Kruskal coordinates, which cover a lot of regions,

$$X^- = lnU, \quad X^+ = -ln(-V) \tag{2.3}$$

[3]In S_{BH}, BH stands for Bekenstein-Hawking, but what a coincidence that one can use BH for Black Holes as well.

[4]For $d\tau^2$, we multiply with a minus to metric.

and

$$r = \frac{1+UV}{1-UV} \tag{2.4}$$

and the metric becomes

$$ds^2 = \frac{1}{(1-UV)^2}\left(-4dUdV + (1+UV)^2 d\Omega^2\right) \tag{2.5}$$

this covers all the regions. For a discussion of various coordinates, see Ref. [13].

At this point, we need to introduce Penrose diagrams. These diagrams are helpful in understanding the causal structures of space-time. A Minkowski space is a flat space with a Penrose diagram as given in Fig. 1. In the diagram, we have null (light-like) geodesics travel at 45° angles to $r = 0$. We have $t = \infty$ as time-like future infinity, $t = -\infty$ as time-like past infinity, and $r = \infty$ as space-like infinity. Likewise, there are Penrose diagrams for Kruskal coordinates space. Well, now the question is: how are black holes formed? These are formed by the collapse of the infalling shell of matter. The Schwarzchild black holes, which we often discuss, are not real black holes we find in the cosmos.

A simple way of constructing a black hole in space would be the collapse of the infalling shell of matter. The Penrose diagram of such an event is Fig. 3(b), where the colored lines are shells of matter that collapses. Birkhoff's theorem states that space is separated by infalling matter. We call the space below, 'A' and the space above 'B'. The part outside the horizon is called the exterior of the black hole, and the space inside the horizon is called the interior.

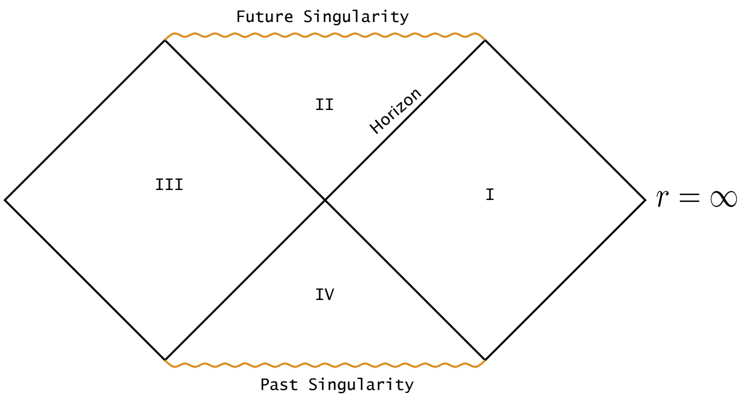

Figure 2. Two-sided extended Penrose diagram for Schwarzchild black hole.

To find out 'B,' we need to move on to the Schwarzchild metric and look at the interior as one of those Schwarzchild interiors. Schwarzchild black hole interior is Fig. 3(b). We can draw a Penrose diagram for the final black hole from collapsing shell of infalling matter, which is Fig. 4.

Schwarzchild BHs, in general and in AdS, are of theoretical importance too. We will come to the possible discussions of the two-sided features of black holes in AdS later. But we must mention that they have been the active candidate for 'wormholes' [14]. However, such wormholes are not traversable, as we will see. See [15,16] for discussions on traversable wormholes.

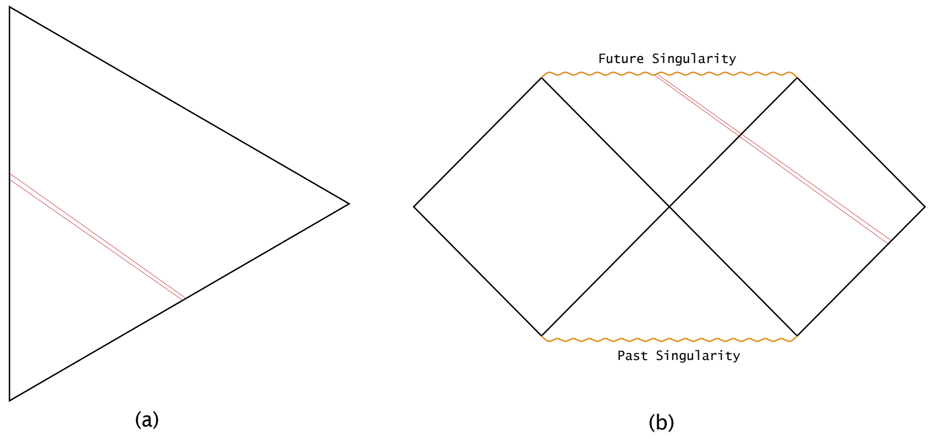

Figure 3. An infalling shell of matter collapses in flat space.

But, *this is not as simple as that.* The interior of the black hole, the region behind the horizon, and singularity are parts of stubborn BH physics. There are many ways to discuss the interior, which we will see in these notes, but none of them completely solves the information problem. However, solving the problem, while being the main problem, does not prevent us from developing physics and mathematics while we go along. Complimentary, unitarity, interior, singularity, entanglement, entropy, observer, cloning, teleportation, radiation, firewalls, fuzzballs, and of course, AdS/CFT are some of the words that we use in this article.

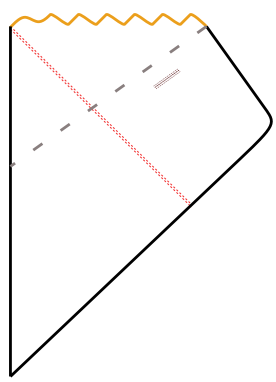

Figure 4. Penrose diagram for collapsing shell of massless particles. The greyish dotted line represents the horizon, and the yellow wavy line represents the singularity. The blue lines going to future infinity represent the Hawking radiation; see Sec. 3.

3 Thermodynamics and Radiation

There have been many attempts at describing the BH horizon and singularity. The Schwarzchild solution is the classic example of a black hole [17]. For an early review of classical solutions of black holes, [1]. In the 1970s, Hawking came up with the idea of the evaporation of black holes and the creation of pairs at the horizon [11]. Moreover, Bekenstein was the first person to actually write down the relationship between black hole and thermodynamics, and also generalized the second law of thermodynamics [18], see also [19–21]. Bekenstein argued that the entropy of the black hole is related to the area of the horizon.

Entropy is the measure of the uncertainty of knowing the configurations of a certain system. Entropy and information theory are related. A very simple case is to consider a box of gas. At some time t, the entropy of the box is defined as the uncertainty of configurations of the state. If we decide to know about the positions of molecules (one of the configurations), then we gain information and hence the entropy is reduced. This implies that if the information is lost throughout a process, entropy must increase as a result. Another way of seeing is that if we know everything about a system, then the entropy of the system is zero.

Hawking theorem's [19] had proved that area of the horizon could not decrease. In the case of two black holes merging, the resulting area is always greater than the sum of the area of individual black holes[5]. Bekenstein proposed that the entropy of black hole, as a thermodynamic system, should only increase as we indicated in Sec. 2. He proposed a solution, which was perfected by Hawking, that the black hole entropy (the physical entropy) should be given by

$$S_{BH} = \frac{A}{4G}, \quad \hbar = c = k_B = 1 \tag{3.1}$$

where G is Newton's constant. It is quite absurd to see that black holes entropy does not require one to investigate the interior of the black holes. For a Schwarzchild BH, area is written as

$$A = 4\pi R^2 = 16\pi M^2 G^2 \tag{3.2}$$

and the S_{BH} is

$$S_{BH} = 4\pi M^2 G \tag{3.3}$$

which is a large number. For instance, a solar mass has entropy $S_{BH} \approx 10^{77}$. This much entropy implies how difficult it is to find the initial conditions of black holes or how large number of microstates a black hole has. Every time, something is tossed into a black hole, the entropy increase and thus the area. Suppose we have a object with entropy S in the visible universe, we can extract the information about the object in time before it falls into a black hole. But once it falls into a black hole, no information is accessible, and the entropy of the visible universe decreases with the object falling into the horizon. This violates the second law. Nonetheless, the entropy gain in black hole is S or more than S.

To compensate the loss of entropy in the outside universe, Bekenstein proposed 'Generalized Second Law of Thermodynamics (GSL) [18, 22]. He argued that the generalized entropy

$$S_G = S_{BH} + S_{out} \tag{3.4}$$

[5]However, the energy is lost in terms of gravitational waves.

can never decrease. The gain in entropy to BH can be compensated from S_{out} and no loss of entropy can be seen in S_G. There were many thought experiments with GSL, by Bekenstein himself [18] and others, well presented in review [23].

In fact, this was not the only law of thermodynamics which appeared in black holes physics. Bardeen, Carter, and Hawking [24] pointed relevant correspondence between thermodynamics and black holes physics. It came as a result that surface gravity κ and temperature are related, analogous to that relation of entropy and area. With all those success, Bekenstein in 1981 [25] also proposed an entropy bound (for universally weakly self-gravitating) with a help of Gedanken experiment. The entropy bound is for matter systems with energy E and radius $\mathcal{R}$

$$S \leq 2\pi E\mathcal{R} \tag{3.5}$$

where E is the total mass-energy of the matter system and $\mathcal{R}$ is the radius of the smallest sphere to fit in the matter system. Note that in Bekenstein entropy Eq. (3.5), with all restored constants, G does not appear and thus one can believe that Bekenstein bound is applicable for other quantum fields as well. This entropy bound is remarkable and had fueled a lot of debates from either side, which only settled with a full proof by Casini [26] for validity of the entropy bound in quantum field theory. Bekenstein and colleagues with [27–36] favored the conjecture and some [37–40] proposed counterexamples on the validity of the bound. Meanwhile, a lot of work happened in entropy bounds, for instance 'Covariant Entropy' bound [35] of which Bekenstein bound was a general limit. See Ref. [33] for some of developments.

We now refer our attention to what Hawking claimed in [11]. This was a terrific idea which still haunts researchers. Hawking believed that there is a positive flux of particles going outward from horizon to future infinity and a negative flux of particles going inward to horizon. This implies that the black holes have some temperature T_H, which we call Hawking temperature now, and is given for Schwarzchild black hole by[6]

$$T_H = \frac{1}{M8\pi G} \tag{3.6}$$

where k_B is Boltzmann constant. For a lower mass black hole, temperature would be high as Eq. (3.6) suggests. (It is $10^{49}K$ for a proton mass black hole.) Eq. (3.6) gives the Bekenstein-Hawking entropy Eq. (3.1) when we solve

$$dM = T\ dS \tag{3.7}$$

this is how the constant of Bekenstein's proposal of black hole was found. That is why the entropy is called Bekenstein-Hawking entropy.

It is surprising, however not unexpected, to learn that black holes create particles, as first observed in [11]. Parker had observed particle creations, in rather slow rate, for an expanding universe [41–43]. We must note that in classical physics, black holes never evaporate because radiations are a consequence of quantum effects near horizon like Bogoliubov transformations. One need not to worry about evaporation at all in classical physics.

[6]It must be noted that if $\hbar$ and c is restored, then this formulae contains all the relevant constants of physics.

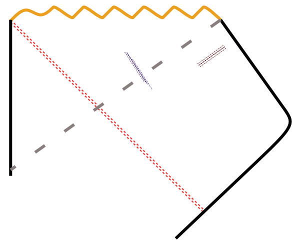

Figure 5. Hawking pairs are created in vacuum, positive flux goes to future infinity and negative flux enters the horizon.

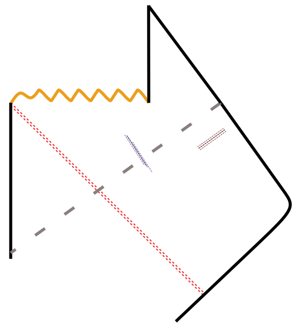

Figure 6. Penrose diagram for evaporating black hole.

Negative flux of particles are generated to preserve the total energy. It is not hard to accept that negative flux will decrease the mass of black holes as suggested by Hawking [11]. The emission rate is given by

$$N_{jwlmp} = \Gamma_{jwlmp} \left(exp \left(2\pi k^{-1} \left(w - m\omega - q_i\Phi \right) \right) \mp 1 \right)^{-1} \tag{3.8}$$

where j represents the species of charge q particles emitted with frequency w, spheroidal harmonic l, and angular momentum m, the minus sign is for bosons and the plus sign for fermions; Ω is surface angular frequency and Φ is surface electrostatic potential. Coefficient Γ_{jwlmp} is the absorption probability for an incoming wave of the mode.

We also must mention the first law of thermodynamics here which relates κ, Φ, Ω, m and A

$$dM = \frac{\kappa}{4\pi} dA + \Omega dJ + \Phi dQ \tag{3.9}$$

from where one can easily understand that $T = \kappa/2\pi$. We refer [11] for the calculation of this formulae, i.e. Eq. 3.8. This emission is dangerous, because it means that black holes evaporate. One can calculate the evaporation time[7] of black holes, as Page has done explicitly in Ref. [44], which is $t_{\text{evp}} \sim M^3G^3$. What happens when a black holes fully evaporates? This has main two answers out of many;

1) *Remnants:* One can suspect that evaporation stops when black holes becomes Planck sized and quantum gravity becomes very important. Then the remaining black hole remnant is all that we have leftover with to recover our information. This goes beyond the

[7] A solar mass black holes evaporates in 10^{67} years. For comparison, the universe is only 13.8 billions years old.

laws of physics; the entanglement entropy becomes larger than the Bekenstein-Hawking entropy at Planckian time. This remnant model, however, does not violate the unitarity sanctity as one gets pure states at the end. But that comes only with remnants having extraordinary entanglement entropy.

2) *Mixed States:* Another possibility is that black holes will completely evaporate, leaving only radiation with mixed states because of the entanglement between Hawking radiation and black hole. These radiations are basically photons, gravitons, and neutrinos (presumably also electrons and positrons when black holes become very small) [44]. However, this mixed state idea violates the unitarity of quantum mechanics as a black hole starts with a pure state. This was endorsed by Hawking initially, but this implies that information is lost in such evolution of black holes. However, while some believe that this is a bad physics and leads to energy conservation violation [45], there are also arguments in favor of energy conservation during evaporation [46]. But we do not discuss either in this paper.

These direct us to paradoxes only and further questioning. One can ignore the issues till the Planck scale, as one in the remnants model does, but that also creates further issue, as we saw, of large entropy inside the remnants. Some have tried to break the semi-classical pictures even before the Planck scale. This contains radical yet informative proposals like AMPS Firewalls [47] and Fuzzballs [48]. So clearly, the answer is not clear but one can be prudent that the answer contains new physics from which we can learn terrific and terrible things about quantum gravity. The affairs related to information loss that we presented come together and are called *Black Hole Information Problem.*

4 Defending Information

Information loss is a major proposal which directly attacks the validity of unitarity in quantum gravity. But we do not want such questions to hang around in the air until we find the most correct description of quantum gravity. Researchers have attempted many ways to defend information, some have even tried to tweak quantum mechanics. Thus far, the works in the black holes information problem have contributed many physics and should continue to do so. The idea of motivation is to understand the profiles of quantum gravity and black holes without having a perfect quantum gravity in hand. There have been attempts to understand black holes after Hawking's proposal which claim to somewhat or fully resolve the information loss. We will discuss some of them in this section.

4.1 Page Curve

What if we could believe that Hawking radiation is a unitary process? For that matter, what are the conditions for the Hawking process to be a unitary evolution of black holes. Page has given an interesting picture which can tell us how black holes can preserve the information unitarily [49–51]. Page's idea was based on his calculations of black holes evaporation [44, 52]. However, the black hole would be assumed to be non-rotating and uncharged to prevent further entanglement and Page assumed that black holes emit massless particles, i.e. photons and gravitons [44]. As we have discussed before, black holes

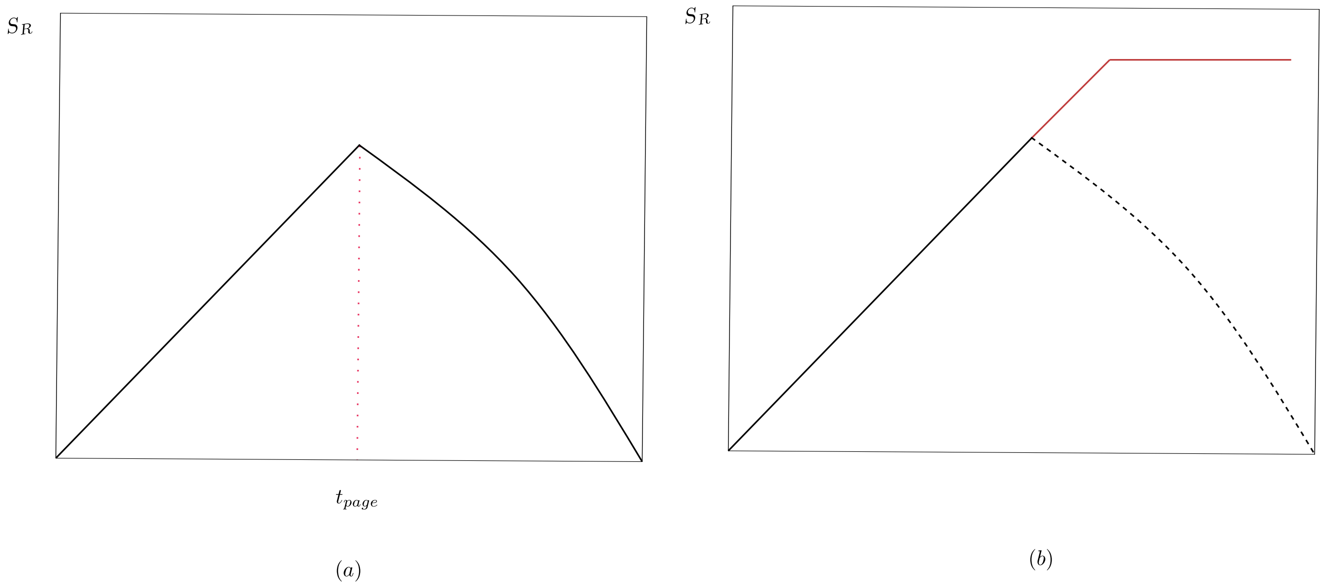

Figure 7. Page curve for a black hole. Semi-classic entropies $S_{BH} = S_R$ at the t_{page} which is page time (about the order Schwarzchild radius r^3).

emission cause black holes' entropy to decrease by the rate $8\pi\alpha/M$ in Planck units, where α is given by

$$\alpha \equiv -M^2 \frac{dM}{dt} \tag{4.1}$$

which has been computed explicitly in [44]. The total decrease in entropy, given $\alpha = 0.000037474$ [44], is given by $-dS_{BH}/dt = 8\pi\alpha/M \approx 0.00094182/M$. This gives the ratio of increase in the radiation entropy to the decrease in the black hole entropy and is calculated

$$\beta = \frac{dS_R/dt}{dS_{BH}/dt} \approx 1.48472 \tag{4.2}$$

which one should clearly see is not unity; we will come again to this point in a while.

Let us now see what Page curve Fig. 7(a) means. We plot the S_R which is the hawking radiation (or entanglement entropy). At the start, we expect $S_R = 0$ for a pure state, then at Page time we can expect the coarse grained semi-classical entropies $S_R = S_{BH}$ and then at the end $S_R = 0$ so that the outside is pure. We must note that it should not be taken that at t_{page} black hole is half-radiated because as noted above $\beta \approx 1.48472$ ($\beta \neq 1$). In fact, $t_{\text{page}} \approx 0.54 t_{\text{evp}}$. In contrast, Fig. 7(b) is what Hawking expected, S_R entanglement entropy continues to increase and then saturates; it never comes back to a pure state and this is how it violates the unitarity.

So if we could draw the Page curve for Hawking evaporation, then we could solve the black hole information problem. At least, it could be proved that information remains preserved in pure state at the end.

4.2 Complementarity

Black holes complementarity[8] is set of postulates given by Susskind, Thorlacius, and Uglum [54, 55]. The postulates are as following

- **Postulate 1:** For a distant observer, the formation and evaporation of the black hole can be described in the context of quantum field theory. In particular, there exists a unitary S-matrix which describes the evolution from infalling matter to outgoing Hawking-like radiation.
- **Postulate 2:** Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations.
- **Postulate 3:** To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole of mass M is the exponential of the Bekenstein entropy $S(M)$.
- **Postulate 4:** A freely falling observer experiences nothing out of the ordinary when crossing the horizon.

4.3 The Firewall Paradox

Complementarity might not be enough; this was pointed out by Almheiri, Marolf, Polchinski, and Sully (also called 'AMPS') [47]. AMPS argues that the horizon becomes a firewall of violent modes when a black hole crosses its scrambling time or, in other words, when black holes become old. It can be understood by first understanding 'monogamy of entanglement', meaning that one system can not be maximally entangled with two different systems simultaneously. Consider Fig. 8 where a slice has three points on it A, B, C in which C can be seen far away from the horizon, a place for early radiation from the black hole, B is close to the horizon and A is on another side of the horizon. If the horizon is meant to be a simple vacuum as of Minkowski's space, then A and B are entangled as one expects [56]. (Another way to say this is that Minkowski's vacuum can be divided into left and right Rindler wedges, which are maximally entangled with each other in any general QFT.) In fact, this is the problem, as we will see later, considering the horizon a simple vacuum does not help solve Hawking's problem, at least for AMPS. Moreover, the maximally entanglement between A and B , $S_{AB} = 0$, is another way of quoting the equivalence principle, which means that an infalling observer notices nothing out-of-ordinary about the passing horizon.

In Fig. 8, A and B are maximally entangled when black holes are young since there is no radiation leaking. But as a black hole emits radiations B and C become maximally entangled because unitarity follows from B to C. However, this creates a paradox as B cannot be maximally entangled to both A and C at one time because of the subadditivity of entropies. So we must abandon either $S_{AB} = 0$ or $S_{BC} = 0$. For sure, the latter cannot

[8]Ref. [53] calls this traditional complementarity and notes that fuzzball complementarity are modified version of fuzzballs argument.

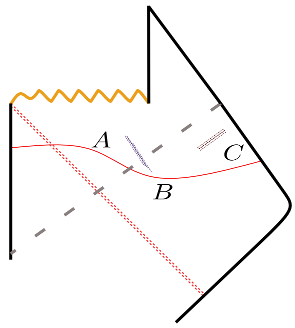

Figure 8. A,B,C are three regions on a slice that we have drawn in red. A is the region inside the horizon, B is the region just outside the horizon, and C is the region far away.

be abandoned in the name of unitarity, and this leaves us with opting out the postulate 4 from complementarity that an infalling observer does not experience special physics at the horizon. The horizon turns into a high-energy firewall where the observer must meet with the end.

A straightforward solution to the firewall paradox is that we have two observers; Alice and Bob. We set Bob to measure the system C and then B ; in this way, he can be sure about the von-Neumann entropy of system BC. We have Alice, then. She can jump inside the black hole after measuring B . Inside the black hole, she measures A , and she gets the von-Neumann entropy of AB. Now let us assume it is too late for Alice to tell Bob about AB. In this way, there is no communication between local observers, and no contradiction is born. However, this is naive as the paradox is still there in the global picture [57], and it fails the consistency checks.

When do firewalls emerge? This was a confusing question as it could be scrambling time ($r_S \log r_S$), or it could be Page time. AMPS [47] argued originally for scrambling time. By Page time, the entanglement between the late radiation and B increases, so almost all black holes that have crossed Page time (which is a big number) have firewalls. In this sense, only young black holes offer any chance of survival before falling into the singularity. It is interesting to mention that with the firewalls on hand, one can say the singularity evolves as it was mentioned by Susskind [58].

5 Wormholes and ER=EPR

Interestingly, black holes also offer us a study about transferring information from one exterior space to another. It has been called 'Wormhole'. Wormholes are the two-sided solution of a black hole. As the name suggests, a two-sided black hole has two exterior spaces rather than one, which we get from the formation of collapsing matter. A simple two-sided solution is Fig. 2, where we have a left exterior space (III) and a right exterior space (I) from an upfront global point of view. Both sides are maximally entangled with each other. That takes us to believe that even one-sided black holes (which we have been discussing in this article) can be thought of as two-sided, with one side being radiation. This

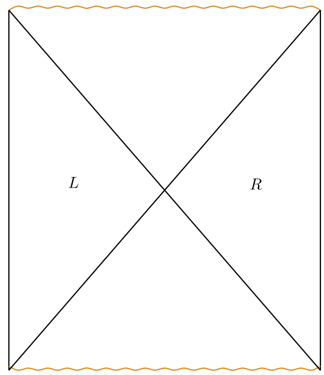

Figure 9. A Penrose diagram of two-sided eternal AdS black hole.

can generally happen after Page time when black holes and early radiation are maximally entangled [59]. For an early review of wormholes, see [60].

Maldacena and Susskind proposed a way to understand the interiors of the black hole by studying two-sided black holes under entanglement [59]. Einstein and Rosen (ER) [14] proposed a black hole with two sides, and Einstein, Podolsky, and Rosen (EPR) [61] wrote famously on entanglement. The researchers in [59] tell us that $ER = EPR$, which means entanglement can lead to Einstein-Rosen bridges. This has been called the $ER = EPR$ conjecture.

Think of a region with two points far away from each other. We take an entangled qubit, and we take one bit to location A and another at location B. Both locations are now entangled in a manner that they create a bridge between points which is far less than the distance between them in exterior space. This bridge is a wormhole. Technically, a qubit is sufficient to create a wormhole, but we need a way to entangle two regions maximally. Location A has a black hole horizon connecting to location B horizon, at least at $t = 0$. As time goes on, the distance between bridges changes, as we see in the Penrose diagram.

The growth of bridges also depends on the entanglement between the regions [62]. We believe that this is one of the reasons behind the non-traversability of the wormhole, that it pinches off and pulls apart from each other so that no observer can cross it—the more the entanglement, the less the distance.

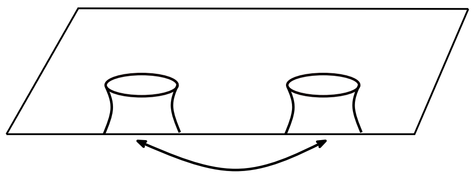

Figure 10. Einstein-Rosen bridge connecting two locations at a later time in Penrose diagram.

It might first seem that EPR and ER violate locality, but that is not true. Entanglement does not violate locality, and Einstein-Rosen bridges can neither provide a way from one exterior space to another.

Let us consider Alice and Bob living in either exterior space of a two-sided black hole. If

Alice and Bob could communicate (that would be a superluminal way in this case) through the wormhole that they have created using entanglement, then it would be the fastest way of communication. If Alice has a message for Bob, then she sends it through the horizon, and Bob has to jump into the horizon to get it. If both do not fall into their respective black holes at the right time, then they may not meet in the interior before collapsing into a singularity. If Alice wishes, then she can create a firewall for Bob, which Bob can only know of if he decides to jump into horizon [59]. There is no way of knowing it beforehand for Bob.

It is interesting that we can create bridges for one-sided black holes as well. A black hole formed by a collapse of matter evaporates, and Hawking radiation is created from an evaporating black hole. We know that early radiation and the Black hole is maximally entangled at Page time, so we can say this entanglement must result in bridges. See Ref. [59] for original arguments.

Traversable wormholes have gained discussions recently, see [15, 63].

We have come to the last of the notes. We have discussed major works, but we could not cover all of them. That leaves us not discussing AdS/CFT as a solution for unitary evolution, baby universes, Ryu-Takayanagi description, and other alternatives to information loss. See Ref. [2] for some discussions. In these notes, we have seen that there are a plethora of ideas, but none of them exactly solves the information problem. Maybe there are still ideas underneath the stones that could perfectly solve the problem. A major idea is needed. However, solutions given in this article and mentioned elsewhere have deepened our knowledge. Black holes can tell us about expanding universe and cosmological constants too. It is indeed that what Bekenstein and Hawking had started will still go on for years. The experimental verification is tedious and sometimes beyond our reach. We have seen many overlaps throughout the years, one of them is string theory-black holes crossover, and we expect to see other crossovers in the future.

A Black Holes as Mirrors

What is the way to extract information from an evaporating black hole? Hayden and Preskill have argued that we can get information from an old black hole [8]. We take a black hole that has radiated by half (in naive words-a black hole which has crossed Page time). Assuming that the black hole evaporation is unitarity and the receiver can control the Hawking radiation in any way they want.

Fig. 11 is Hayden-Preskill's way of extracting information from an evaporating black hole. If Alice has message M maximally entangled with a reference system M_R and she wishes to send it to Bob, then she can throw it out in an old black hole and wait for the thermalization of qubits. That is a swift process in order to avoid cloning. Now Bob has a system B maximally entangled to B_R. V is a unitary transformation. Then the radiation R is emitted. Dimensions of R should be bigger than that of E. Now Bob has subsystem

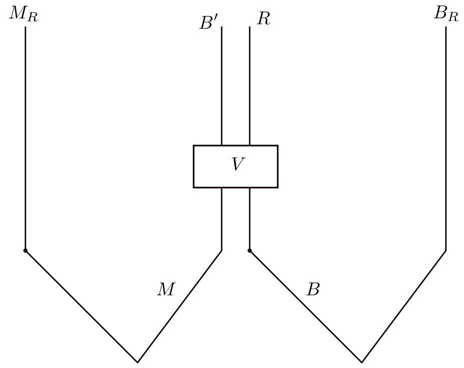

Figure 11. Hayden-Preskill thought experiment.

RE, which is nearly maximally entangled with M_R, which he can feed into the computer to know Alice's message.

Can Bob jump into the black hole after knowing Alice's message and then observe Alice's message inside the black hole too? The answer should be no. We can say that the process is too fast for Bob to ever meet Alice inside the black hole. This was put by Hayden, and Preskill [8]. An interesting development was put by Ref. [63] where AdS is used for the Hayden-Preskill experiment. Ref. [63] also mentions information transfer through wormholes.

References

[1] R. M. Wald, *General relativity*. University of Chicago press, 2010.

[2] D. Harlow, "Jerusalem Lectures on Black Holes and Quantum Information," *Rev. Mod. Phys.* **88** (2016) 015002, arXiv:1409.1231 [hep-th].

[3] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, "The entropy of Hawking radiation," *Rev. Mod. Phys.* **93** no. 3, (2021) 035002, arXiv:2006.06872 [hep-th].

[4] S. Raju, "Lessons from the information paradox," *Phys. Rept.* **943** (2022) 1–80, arXiv:2012.05770 [hep-th].

[5] A. Strominger, "Five Problems in Quantum Gravity," *Nucl. Phys. B Proc. Suppl.* **192-193** (2009) 119–125, arXiv:0906.1313 [hep-th].

[6] E. Witten, "A Mini-Introduction To Information Theory," *Riv. Nuovo Cim.* **43** no. 4, (2020) 187–227, arXiv:1805.11965 [hep-th].

[7] A. Verma, "Notes on Information Theory,". `https://aayushayhverma.files.wordpress.com/2022/03/information_theory.pdf`.

[8] P. Hayden and J. Preskill, "Black holes as mirrors: Quantum information in random subsystems," *JHEP* **09** (2007) 120, arXiv:0708.4025 [hep-th].

[9] J. Preskill, "Do black holes destroy information?," in *International Symposium on Black holes, Membranes, Wormholes and Superstrings*. 1, 1992. arXiv:hep-th/9209058.

[10] D. Gottesman, A. Kitaev, and J. Preskill, "Encoding a qubit in an oscillator," *Phys. Rev. A* **64** (2001) 012310, arXiv:quant-ph/0008040.

[11] S. W. Hawking, "Particle Creation by Black Holes," *Commun. Math. Phys.* **43** (1975) 199–220. [Erratum: Commun.Math.Phys. 46, 206 (1976)].

[12] S. W. Hawking, "Black holes in general relativity," *Commun. Math. Phys.* **25** (1972) 152–166.

[13] L. Susskind and J. Lindesay, *An introduction to black holes, information and the string theory revolution: The holographic universe*. 2005.

[14] A. Einstein and N. Rosen, "The Particle Problem in the General Theory of Relativity," *Phys. Rev.* **48** (Jul, 1935) 73–77. `https://link.aps.org/doi/10.1103/PhysRev.48.73`.

[15] P. Gao, D. L. Jafferis, and A. C. Wall, "Traversable Wormholes via a Double Trace Deformation," *JHEP* **12** (2017) 151, arXiv:1608.05687 [hep-th].

[16] M. S. Morris and K. S. Thorne, "Wormholes in space-time and their use for interstellar travel: A tool for teaching general relativity," *Am. J. Phys.* **56** (1988) 395–412.

[17] K. Schwarzschild, "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin* (Jan., 1916) 189–196.

[18] J. D. Bekenstein, "Black holes and entropy," *Phys. Rev. D* **7** (1973) 2333–2346.

[19] S. W. Hawking, "Gravitational radiation from colliding black holes," *Phys. Rev. Lett.* **26** (1971) 1344–1346.

[20] D. Christodoulou, "Reversible and irreversible transforations in black hole physics," *Phys. Rev. Lett.* **25** (1970) 1596–1597.

[21] R. Penrose and R. M. Floyd, "Extraction of rotational energy from a black hole," *Nature* **229** (1971) 177–179.

[22] J. D. Bekenstein, "Generalized second law of thermodynamics in black hole physics," *Phys. Rev. D* **9** (1974) 3292–3300.

[23] R. M. Wald, "The Thermodynamics of Black Holes," *NATO Sci. Ser. II* **60** (2002) 477–522.

[24] J. M. Bardeen, B. Carter, and S. W. Hawking, "The Four laws of black hole mechanics," *Commun. Math. Phys.* **31** (1973) 161–170.

[25] J. D. Bekenstein, "A Universal Upper Bound on the Entropy to Energy Ratio for Bounded Systems," *Phys. Rev. D* **23** (1981) 287.

[26] H. Casini, "Relative entropy and the Bekenstein bound," *Class. Quant. Grav.* **25** (2008) 205021, arXiv:0804.2182 [hep-th].

[27] J. D. Bekenstein, "Energy Cost of Information Transfer," *Phys. Rev. Lett.* **46** (1981) 623–626.

[28] J. D. Bekenstein, "BLACK HOLES AND EVERYDAY PHYSICS," *Gen. Rel. Grav.* **14** (1982) 355–359.

[29] J. D. Bekenstein, “ENTROPY BOUNDS AND THE SECOND LAW FOR BLACK HOLES,” *Phys. Rev. D* **27** (1983) 2262–2270.

[30] M. Schiffer and J. D. Bekenstein, “Proof of the Quantum Bound on Specific Entropy for Free Fields,” *Phys. Rev. D* **39** (1989) 1109–1115.

[31] J. D. Bekenstein, “Holographic bound from second law of thermodynamics,” *Phys. Lett. B* **481** (2000) 339–345, arXiv:hep-th/0003058.

[32] J. D. Bekenstein, “Limitations on quantum information from black hole physics,” *Acta Phys. Polon. B* **32** (2001) 3555–3570, arXiv:quant-ph/0110005.

[33] R. Bousso, “The Holographic principle,” *Rev. Mod. Phys.* **74** (2002) 825–874, arXiv:hep-th/0203101.

[34] J. D. Bekenstein, “How does the entropy / information bound work?,” *Found. Phys.* **35** (2005) 1805–1823, arXiv:quant-ph/0404042.

[35] R. Bousso, “A Covariant entropy conjecture,” *JHEP* **07** (1999) 004, arXiv:hep-th/9905177.

[36] R. Bousso, *Black hole entropy and the Bekenstein bound*. 2020. arXiv:1810.01880 [hep-th].

[37] D. N. Page, “Comment on a Universal Upper Bound on the Entropy to Energy Ratio for Bounded Systems,” *Phys. Rev. D* **26** (1982) 947.

[38] W. G. Unruh and R. M. Wald, “Acceleration Radiation and Generalized Second Law of Thermodynamics,” *Phys. Rev. D* **25** (1982) 942–958.

[39] W. G. Unruh and R. M. Wald, “ENTROPY BOUNDS, ACCELERATION RADIATION, AND THE GENERALIZED SECOND LAW,” *Phys. Rev. D* **27** (1983) 2271–2276.

[40] D. N. Page, “Huge violations of Bekenstein’s entropy bound,” arXiv:gr-qc/0005111.

[41] L. Parker, “Particle creation in expanding universes,” *Phys. Rev. Lett.* **21** (1968) 562–564.

[42] L. Parker, “Quantized fields and particle creation in expanding universes. 1.,” *Phys. Rev.* **183** (1969) 1057–1068.

[43] L. Parker, “Quantized fields and particle creation in expanding universes. 2.,” *Phys. Rev. D* **3** (1971) 346–356. [Erratum: Phys.Rev.D 3, 2546–2546 (1971)].

[44] D. N. Page, “Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole,” *Phys. Rev. D* **13** (1976) 198–206.

[45] T. Banks, L. Susskind, and M. E. Peskin, “Difficulties for the Evolution of Pure States Into Mixed States,” *Nucl. Phys. B* **244** (1984) 125–134.

[46] W. G. Unruh and R. M. Wald, “Information Loss,” *Rept. Prog. Phys.* **80** no. 9, (2017) 092002, arXiv:1703.02140 [hep-th].

[47] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, “Black Holes: Complementarity or Firewalls?,” *JHEP* **02** (2013) 062, arXiv:1207.3123 [hep-th].

[48] S. D. Mathur, "The Fuzzball proposal for black holes: An Elementary review," *Fortsch. Phys.* **53** (2005) 793–827, arXiv:hep-th/0502050.

[49] D. N. Page, "Information in black hole radiation," *Phys. Rev. Lett.* **71** (1993) 3743–3746, arXiv:hep-th/9306083.

[50] D. N. Page, "Time Dependence of Hawking Radiation Entropy," *JCAP* **09** (2013) 028, arXiv:1301.4995 [hep-th].

[51] D. N. Page, "Average entropy of a subsystem," *Phys. Rev. Lett.* **71** (1993) 1291–1294, arXiv:gr-qc/9305007.

[52] D. N. Page, "Particle Emission Rates from a Black Hole. 2. Massless Particles from a Rotating Hole," *Phys. Rev. D* **14** (1976) 3260–3273.

[53] S. D. Mathur and D. Turton, "The flaw in the firewall argument," *Nucl. Phys. B* **884** (2014) 566–611, arXiv:1306.5488 [hep-th].

[54] L. Susskind, L. Thorlacius, and J. Uglum, "The Stretched horizon and black hole complementarity," *Phys. Rev. D* **48** (1993) 3743–3761, arXiv:hep-th/9306069.

[55] L. Susskind and L. Thorlacius, "Gedanken experiments involving black holes," *Phys. Rev. D* **49** (1994) 966–974, arXiv:hep-th/9308100.

[56] W. G. Unruh, "Notes on black hole evaporation," *Phys. Rev. D* **14** (1976) 870.

[57] R. Bousso, "Complementarity Is Not Enough," *Phys. Rev. D* **87** no. 12, (2013) 124023, arXiv:1207.5192 [hep-th].

[58] L. Susskind, "Singularities, Firewalls, and Complementarity," arXiv:1208.3445 [hep-th].

[59] J. Maldacena and L. Susskind, "Cool horizons for entangled black holes," *Fortsch. Phys.* **61** (2013) 781–811, arXiv:1306.0533 [hep-th].

[60] M. S. Morris and K. S. Thorne, "Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity," *American Journal of Physics* **56** no. 5, (1988) 395–412.

[61] A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Phys. Rev.* **47** (May, 1935) 777–780. `https://link.aps.org/doi/10.1103/PhysRev.47.777`.

[62] M. Van Raamsdonk, "Building up spacetime with quantum entanglement," *Gen. Rel. Grav.* **42** (2010) 2323–2329, arXiv:1005.3035 [hep-th].

[63] J. Maldacena, D. Stanford, and Z. Yang, "Diving into traversable wormholes," *Fortsch. Phys.* **65** no. 5, (2017) 1700034, arXiv:1704.05333 [hep-th].