On Unparticle Physics and Its Interactions

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Abstract

In the following discussion, we study the literature of Unparticle Physics and its interaction with high energy physics in the presence of the standard model, for instance, the mono-photon process and mono-Z production. We also review the propagators of unparticle regimes.

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1 Introduction and Discussion

Unparticle Physics, which was introduced to propose unnatural and unseen scale-invariant interactions in high energy physics, which converges at the Standard Model (SM) energy, has quite physics in it right now. They usually do not interact with SM particles in a usual manner. It was proposed by Georgi [1, 2] in 2007. The most (and the lead) attractive point of the theory is scale invariance. Scale invariance implies that a theory with a scale-invariant lagrangian would not change by changing the theory's scale; for instance, a massless scalar field is scale-invariant.

Scale-invariant theories are considered to be conformal invariant theories by default. Scale-invariant stuff mass is believed to be zero, as non-zero masses cannot pose for scale invariance, as they have mass dimensions. The primary example of scale invariance is a massless scalar field. A detailed paper is written already on the bounds of conformal invariance of unparticle operators [3], which we will discuss later.

Unparticle Physics arises from a weakly coupled Bank-Zaks field [4]. Since the derivation, unparticle physics has been explored a lot in many ways [5–8]. We mention here some recent development in unparticle physics, especially theoretical aspects. The paper [1], which introduces the stuff, tries to explain this unparticle scheme. For very high energy, we can say that standard model fields and fields of non-trivial IR fixed point, which we call Banks-Zaks fields ($\mathcal{BZ}$), will interact through a large scale called $M_{\mathcal{U}}$. This $M_{\mathcal{U}}$ suppresses the obvious nonrenormalizable couplings in the coupling when the scale is below $M_{\mathcal{U}}$. For larger $M_{\mathcal{U}}$, the following Eq (1) does not allow the unparticle stuffs to couple with ordinary matter. These couplings, when $\mathcal{BZ}$ and SM fields interact, have been given a form

$$\frac{1}{M_{\mathcal{U}}^{k}} O_{SM} O_{\mathcal{BZ}}, \tag{1}$$

where k = $d_{SM} + d_{\mathcal{BZ}} - 4$ and $\mathcal{BZ}$ gives the $O_{\mathcal{BZ}}$ operator with $d_{\mathcal{BZ}}$ mass dimension and O_{SM} is built out from standard model with d_{SM} mass dimension. Because of the fixed point nature, $\mathcal{BZ}$ has non-trivial scale invariance-which emerges at some energy level $\Lambda_{\mathcal{U}}$-making the renormalizable couplings in $\mathcal{BZ}$ to perform dimensional transmutation[2]. It can be easily seen that $\mathcal{BZ}$ fields can act as $O_{\mathcal{U}}$ fields below the defined $\Lambda_{\mathcal{U}}$ by matching the operators, which then makes the interaction look like

$$C_{\mathcal{U}} \frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}} - d_{\mathcal{U}}}}{M_{\mathcal{U}}^{k}} O_{SM} O_{\mathcal{U}}, \tag{2}$$

where $C_{\mathcal{U}}$ is the coefficient function, and $d_{\mathcal{U}}$ is the scaling dimension of the unparticle operator, and now it will be treated as a scalar property. In particular, $d_{\mathcal{U}}$ represents the number of massless unparticles. We will later see that $d_{\mathcal{U}}$ can be a non-integral number.

Then, for the best low-energy theory, Georgi chooses that our $O_{\mathcal{U}}$ to be of the lowest dimension. The Eq. (1) gives an emergence to our unparticle effective field theory which is actually Eq. (2). Thus, Eq. (2) is the effective field theory which we can interpret as an unparticle physics field. Eq. (2) is no any different from Eq. (1) in terms of IR scale invariance because when $\mathcal{BZ}$ fields decouple from SM field at low-energy, IR scale invariance is preserved.

It is not necessary that probes of unparticle physics can only be found at TeV scales. If the theory is perturbative, then for smaller $M_{\mathcal{U}}$, we can try to probe these kinds of stuff in LHC (or any other collider). We will discuss this in detail later. $\mathcal{U}$ particles must be massless. However, they are believed to have some continuous mass spectra, hence not any invariant fixed mass. There can be solutions to this mass query defined under the CFT boundary concepts, of which we are not sure. It was also noted that these $\mathcal{U}$ energies if found in the collider, will make us understand the missing energy distribution caused by the non-integral value of $d_{\mathcal{U}}$, of which definition we will clear later.

[2] This links to the mass dimensions we are discussing. These dimensional transmutations are easily seen in massless non-abelian gauge theories.

So, what does this theory produces at low-energy below the $\Lambda_{\mathcal{U}}$?

It will produce some irregular forces. That is a very typical phenomenon that must happen. Our paper does not have the scope of those irregular forces. Notably, we mention here that the EFT will produce unparticle stuff at low energy, serving as a QFT and partially contributing to missing energies. This is not such a complicated thing to imagine, as in (2), we mentioned the factors. Moreover, these factors will give us the results we discussed now (missing energies). There is a probability distribution of the interaction, which does require the density of final states. However, for our low-energy scheme, we can overthrow this requirement because of scale invariance. Then our matrix element in a vacuum is [1]

$$\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^{\dagger}(0)|0\rangle = \int e^{-iPx}|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2\rho(P^2)\frac{d^4P}{(2\pi)^4}, \tag{3}$$

where $|P\rangle$ is our unparticle state with 4-momentum P^{μ} born out of the vacuum by $O_{\mathcal{U}}$. Moreover, $\rho(P^2)$ is the spectral density. If we try to impose certain constraints on the given matrix element, then (3) scales with dimension $2d_{\mathcal{U}}$. However, a certain condition applies for that scaling property which is in a good phase for our EFT[3](and this phase serves an important role here),

$$|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2\rho(P^2) = A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2} \tag{4}$$

where $A_{d_{\mathcal{U}}}$ is a normalized factor to interpolate the massless $d_{\mathcal{U}}$-body phase spaces. The powers of the last term are based on scale transformation, which is for dimension $2d_{\mathcal{U}}$. (4) is just the inverse Fourier transformation of spectral density.

The phase space for n massless particles is given by

$$A_n = \frac{16\pi^{5/2}}{(2\pi)^{2n}}\frac{\Gamma(n+1/2)}{\Gamma(n-1)\Gamma(2n)}, \tag{5}$$

and in (4) the phase factor associated with the spectral density is given by (5) with $n \to d_{\mathcal{U}}$

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}, \tag{6}$$

and we see that (6) makes a peculiar observation, which has not been seen, that now $d_{\mathcal{U}}$ can be non-integral, hence fractional, number of massless particles as well [1].

The computable cross-section of two massless particles colliding and producing an unparticle and some other massless particles is given by [7]

$$d\sigma(p_1, p_2 \to P_{\mathcal{U}}, k_1, k_2, k_3, ...) = \frac{1}{2(p_1+p_2)^2}|\bar{\mathcal{M}}|^2 d\Phi \tag{7}$$

where

$$d\Phi = (2\pi)^4\delta^{(4)}\left[(p_1+p_2-(P_{\mathcal{U}}+k_1+k_2+k_3+...)\right]\Pi_i\left[2\pi\theta(k_i)^0\delta(k_i^2)\frac{d^4k_i}{(2\pi)^4}\right]\times\omega \tag{8}$$

[3]The left side of the equation contains an extra $(2\pi)^4$ which comes from (3) while the PDG has not this extra factor.

and

$$\omega = A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2}\frac{d^4P_{\mathcal{U}}}{(2\pi)^4} \tag{9}$$

With the limit $d_{\mathcal{U}} \to 1$

$$\lim_{d_{\mathcal{U}}\to 1} A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2} = 2\pi\theta(P^0_{\mathcal{U}})\delta(P^2_{\mathcal{U}}), \tag{10}$$

and this one tells us that under this limit, which is just a unit limit, the unparticle behaves just like other massless particles with the usual scale invariance.

2 Unparticle Operators and Propagators

A derived virtual propagator for interaction in unparticle physics is too doubting at first, but it becomes reasonable for such field theories as we proceed. We will consider a scalar operator at first. An unparticle propagator must be of a scale-invariant kind, which can be derived from our derived phase factor and spectral equations

$$\Delta F(P^2) = \frac{1}{2\theta}\int\frac{R(M^2)dM^2}{P^2 - M^2} - i\frac{1}{2}R(P^2)\theta(P^2), \tag{11}$$

where we define the $R(M^2)$ as spectral density.

But we want a better idea of propagator in unparticle physics. We have propagators for massless particles such as γ

$$\Delta F(P^2) = \frac{1}{P^2}, \tag{12}$$

and if we impose the scale invariance to the propagator it becomes

$$\begin{aligned}\Delta F(P^2) &= -Z_{d_{\mathcal{U}}}\frac{1}{(P^2)^{-d_{\mathcal{U}}+2}} \\ &= Z_{d_{\mathcal{U}}}(-P^2)^{d_{\mathcal{U}}-2},\end{aligned} \tag{13}$$

$Z_{d_{\mathcal{U}}}$ is mandatory for scale invariance. $Z_{d_{\mathcal{U}}}$ will be found soon in this section. For $P^2 > 0$, we will get no mathematical cuts, so we will adopt for $[-\pi, \pi)$

$$(-P^2)^{d_{\mathcal{U}}-2} \to (P^2)^{d_{\mathcal{U}}-2}e^{-id_{\mathcal{U}}\pi} \tag{14}$$

$Z_{d_{\mathcal{U}}}$ can be found by comparing the imaginary value of (13) and we found that

$$Z_{d_{\mathcal{U}}} = \frac{A_{d_{\mathcal{U}}}}{2sin(d_{\mathcal{U}}\pi)} \tag{15}$$

hence, our unparticle operator is

$$\Delta F(P^2) = \frac{A_{d_{\mathcal{U}}}}{2sin(d_{\mathcal{U}}\pi)}(P^2)^{d_{\mathcal{U}}-2}e^{-id_{\mathcal{U}}\pi} \tag{16}$$

substituting $d_{\mathcal{U}} \to 1$ yields the $Z_{d_{\mathcal{U}}}$ as -1 and giving back our γ propagator which was

$$\frac{1}{P^2} \tag{17}$$

That was for a scalar operator $O_{\mathcal{U}}$. What about covariant form operators? Because, the scalar operator will yield just spin-0 form. However, we can extend this knowledge to spin-1 or perhaps spin-2. In vacuum, one can consider for spin-1 tensor form

$$\langle 0|O_{\mathcal{U}}^{u}(x)O_{\mu}^{v\dagger}(0)|0\rangle = A_{d_{\mathcal{U}}} \int \frac{d^4P}{(2\pi)^4} e^{-iPx}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2}\ k^{\mu\nu}(P) \tag{18}$$

where

$$k^{\mu\nu}(P) = -g_{\mu\nu} + \frac{P^{\mu}P^{\nu}}{P^2} \tag{19}$$

a same formalism that we used for calculating the propagator above can be applied to get the propagator for spin-1 operators.

$$\Delta F(P^2) = \frac{A_{d_{\mathcal{U}}}}{2sin(d_{\mathcal{U}}\pi)}(P^2)^{d_{\mathcal{U}}-2}e^{-id_{\mathcal{U}}\pi}\ k^{uv}(P) \tag{20}$$

Just alike for tensor propagators

$$\langle 0|O_{\mathcal{U}}^{uv}(x)O_{\mu}^{\eta\sigma\dagger}(0)|0\rangle = A_{d_{\mathcal{U}}} \int \frac{d^4P}{(2\pi)^4} e^{-iPx}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2}\ T^{uv,\eta\sigma}(P) \tag{21}$$

where

$$T^{uv,\eta\sigma}(P) = \frac{1}{2}\left[\pi^{u\eta}(P)\pi^{v\sigma}(P) + \pi^{u\sigma}(P)\pi^{v\eta}(P) - \frac{2}{3}\pi^{uv}(P)\pi^{\eta\sigma}(P)\right] \tag{22}$$

and

$$\Delta F(P^2) = \frac{A_{d_{\mathcal{U}}}}{2sin(d_{\mathcal{U}}\pi)}(P^2)^{d_{\mathcal{U}}-2}e^{-id_{\mathcal{U}}\pi}\ T^{uv,\eta\sigma}(P) \tag{23}$$

We have thought of all unparticle operators as observables. It is also important to note that $P_u k^{uv}(P) = 0$ and $P_u T^{uv,\eta\sigma}(P) = 0$. The operator $O_{\mathcal{U}}^{u}$ and $O_{\mathcal{U}}^{uv}$ are thought to be transverse, and $O_{\mathcal{U}}^{u}$ to be traceless.

2.1 SM Effective Operators

If we incorporate the standard model interactions which satisfy the gauge symmetry, then our scalar unparticle operator, as in [7], is as follows;

$$\begin{aligned} &\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} f O_{\mathcal{U}}, \\ &\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} i \gamma^5 f O_{\mathcal{U}}, \\ &\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma^\mu f (\partial_\mu O_{\mathcal{U}}) \\ &\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} G_{\alpha\beta} G^{\alpha\beta} O_{\mathcal{U}} \end{aligned} \tag{24}$$

and for our vector unparticle operator

$$\begin{aligned} &\lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_\mu f O_{\mathcal{U}}^\mu \\ &\lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_\mu \gamma_5 f O_{\mathcal{U}}^\mu \end{aligned} \tag{25}$$

for tensor like unparticle operator, we define it as follows

$$\begin{aligned} -\frac{1}{4} \lambda_2 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{\psi} i \left(\gamma_\mu D_\nu + \gamma_\nu D_\mu\right) \psi O_{\mathcal{U}}^{\mu\nu} \\ \lambda_2 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} G_{\mu\alpha} G_\nu^\alpha O_{\mathcal{U}}^{\mu\nu} \end{aligned} \tag{26}$$

where $\lambda_1, \lambda_2, \lambda_3$ are dimensionless couplings in the form of $C^i_{O_{\mathcal{U}}} \Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}} / M_{\mathcal{U}}^{d_{SM}+d_{\mathcal{BZ}}-4}$ and i runs from 0 to 2, which are for scalar, vector and tensor like operators, respectively, and $D_\mu = \partial_\mu + ig\frac{\tau^\alpha}{2} W_\mu^\alpha$ is covariant derivative, denoted f is our standard model fermion, ψ is standard model fermion doublet or singlet and $G^{\alpha\beta}$ is yang-mills gauge field strength. And we assume that our λ_i is flavor blind. These operators and mentioned operators in [5] are very helpful in describing the interesting phenomenology related to unparticles.

In all these SM operators, the lowest dimension of the SM operator is unparticle coupling to two Higgs fields, $H^\dagger H O_{\mathcal{U}}$, which motivates us to find this coupling at low-energies. Moreover, the second-lowest is unparticle coupling with two right-handed neutrinos, $\nu_R^c \nu_R O_{\mathcal{U}}$. Otherwise, every coupling has exact dimensions with SM fields.

The reason for $H^\dagger H O_{\mathcal{U}}$ not observing at low-energies is, as given in [5], that when Higgs field creates a non-zero v.e.v, which in turn is required by gauge symmetry, $\langle H \rangle = v/\sqrt{2}$. This v.e.v is required for SM mass production by Higgs field. There is a coupling of unparticle to v.e.v which is tadpole coupling given by $\lambda_{hh} \Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}} v^2/2$. This interaction deforms the scale-invariance property of unparticles and makes them non-scale invariant, which is then our normal particle physics.

However, there is an attempt to remove the tadpole, if we include another operator $(H^\dagger H)^2 O_{\mathcal{U}}$. This would also have a tadpole coupling, which is $\lambda_{4h}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}$. But if we have

$$\lambda_{4h} v/2 + \lambda_{hh}\Lambda_{\mathcal{U}}^2 = 0 \tag{27}$$

then tadpole is removed. And if we ignore the Goldstone boson that would appear, then

$$\lambda_{hh}\Lambda_{\mathcal{U}}^{2-d_{\mathcal{U}}} H^\dagger H O_{\mathcal{U}} + \lambda_{4h}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}(H^\dagger H)^2 O_{\mathcal{U}} = \frac{1}{4}\lambda_{4h}\Lambda_{\mathcal{U}}^{-d_{\mathcal{U}}}(2v^3 h + 5v^2h^2 + 4vh^3 + h^4) \tag{28}$$

There is an observed mixing between h and $O_{\mathcal{U}}$, which causes the Higgs field's oscillation into $O_{\mathcal{U}}$. This effect then fades away, and the Higgs field gets disappeared. However, the presented mechanism is our assumption of extra 4 Higgs field coupling to unparticle. This has not been observed too. We can hope for these observations only if we have sufficient data related to unparticles, which we have not produced in manners.

The interaction of unparticle with two-right-handed neutrinos is also not observable because ν_R is too heavy as a sterile neutrino, which has a predicted mass up to 1 TeV. Hence, the overall interaction is suppressed for unparticles. However, if ν_R decays to lighter sterile neutrinos, one may observe the effect.

3 Standard Model and Unparticles Interaction

We can note that $d_{\mathcal{U}}$ is playing an essential role in every process. For example, for limit $d_{\mathcal{U}} \to 1$, the particle looks like a regular particle (as we have a standard limit here). Nevertheless, the critical feature is that $d_{\mathcal{U}}$ can have non-integral values. If we apply this non-integral property, then we some get good things in particle phenomenology. If we can think of $d_{\mathcal{U}}$ being the most significant possible fractional renormalizable quantity, then there are many things going on and some of them entirely new. It would be hard to calculate such a process where $d_{\mathcal{U}}$ goes to infinity, but we can guess that it should be related to the N-particles amplitude concept.[4].

Now, we explore some simple interactions in the unparticle regime. Our propagators have been discussed and will be used throughout the overall discussion.

3.1 e^+e^- collision and monophoton

$e^+(p_1)e^-(p_2) \to \gamma(k_1)\ \mathcal{U}(P_{\mathcal{U}})$ and $e^+(p_1)e^-(p_2) \to \gamma(k_1)\ \mathcal{U}(P_{\mathcal{U}})$ interactions contain monophoton which can be used to probe the unparticle [7]. The cross section for $e^+(p_1)e^-(p_2) \to \gamma(k_1)\ \mathcal{U}(P_{\mathcal{U}})$ is

[4] That would be a serious application of this theory.

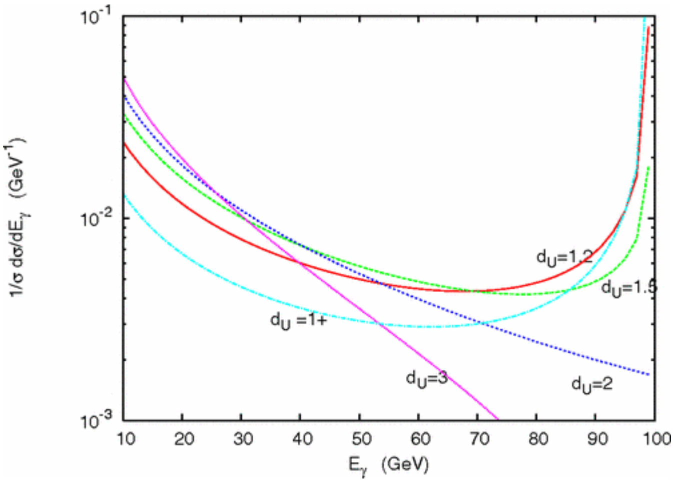

Figure 1: Mono-photon energy spectrum for the process $e^+e^- \rightarrow \gamma\mathcal{U}$ for $d_\mathcal{U} = 1 + \epsilon, 1.2, 1.5, 2$ and $3at\sqrt{s} = 200GeV$. We have imposed $|cos\theta_\gamma| < 0.95$, reproduced from [6].

$$d\sigma = \frac{1}{2s}|\bar{\mathcal{M}}|^2 \frac{A_{d_\mathcal{U}}}{16\pi^3\Lambda_\mathcal{U}^2}\left(\frac{P_\mathcal{U}^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-2} E_\gamma dE_\gamma d\omega \tag{29}$$

where

$$|\bar{\mathcal{M}}|^2 = 2e^2Q_e^2\lambda_1^2\frac{u^2+t^2+2sP_\mathcal{U}^2}{ut} \tag{30}$$

and $P_\mathcal{U}^2$ is related to E_γ, by recoil mass equation

$$P_\mathcal{U}^2 = s - 2\sqrt{s}E_\gamma \tag{31}$$

The monophoton energy for different $d_\mathcal{U}$ will provide insightful data. However, there have been still going searches of monophoton [9]. Using the monophoton data from here and comparing it with dark-matter researches (or any) will help in the probe. The energy spectrum for mono-photon is in fig. (1).

3.2 Mono-Z Production

We also have a mono Z situation here in $f(p)\bar{f}(p') \rightarrow Z(k)\mathcal{U}(P_\mathcal{U})$. For this interaction we have

$$d\sigma = \frac{1}{2s}|\bar{\mathcal{M}}|^2 \frac{\sqrt{E_z^2 - M_Z^2}A_{d_\mathcal{U}}}{16\pi^3\Lambda_\mathcal{U}^2}\left(\frac{P_\mathcal{U}^2}{\Lambda_\mathcal{U}^2}\right)^{d_\mathcal{U}-2} \theta(P_\mathcal{U}^0)\theta(P_\mathcal{U}^2)\; dE_Z\; d\Omega_Z \tag{32}$$

where $|\bar{\mathcal{M}}|^2$ is spin and color averaged and the recoil mass relation is

$$P_\mathcal{U}^2 = s + M_Z^2 - 2\sqrt{s}E_Z \tag{33}$$

where

$$M_Z \leq E_Z \leq E_Z^{\max} = \frac{s + M_Z^2}{2\sqrt{s}}. \tag{34}$$

We define $s + t + u = M_Z^2 + P_{\mathcal{U}}^2$, where s, t, u are Mandelstem variables. And same here when $d_{\mathcal{U}} \to 1$, our unparticle acts as a regular SM scalar. Furthermore, the cross-sections also become a usual case of the standard model. However, at the non-integral limit, this phenomenology is very superior to just a standard process.

3.3 $Z \to f\overline{f}\mathcal{U}$

The decay width of the process $Z \to f\overline{f}\mathcal{U}$ for spin-1 unparticle is given by

$$\frac{d\Gamma(Z \to f\overline{f} + \mathcal{U})}{dx_1 dx_2 d\xi_3} = \Gamma(Z \to f\overline{f} + \mathcal{U})\frac{\lambda_1^2}{8\pi^3} g(1 - x_1, 1 - x_2, \xi)\frac{M_Z^2}{\Lambda_{\mathcal{U}}^2}\left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2} \tag{35}$$

where $\xi = P_{\mathcal{U}}^2/M_{\mathcal{U}}^2$ and $x_{1,2} = 2E_{f,\overline{f}/M_Z}$ is the energy fractions of the fermions involved. The function $g(1 - x_1, 1 - x_2, \xi)$ is defined as

$$g(1 - x_1, 1 - x_2, \xi) = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{(1 + z^2)}{xy} - \frac{z}{2}\left(\frac{1}{x^2} + \frac{1}{y^2}\right) - (1 + z)\left(\frac{1}{x} + \frac{1}{y}\right), \tag{36}$$

and the domain of the (35) is given by $0 < \xi < 1, 0 < x_1 < 1 - \xi$ and $1 - x_1 - \xi < x_2 < (1 - x_1 - \xi)/(1 - x_1)$. As plotted in [6, 7], the plot depends on the scale dimension of unparticle operator. It is easy to see that for $d_{\mathcal{U}} \to 1$, this becomes $\gamma^* \to q\bar{q}g$.

3.4 Mono-Jet and Hadronic Collisions

Partonic led hadronic collision are also important for unparticles phenomenology. It was pointed in the Georgi's paper [1].

$$gg \to g\mathcal{U},\ q\overline{q} \to g\mathcal{U} \tag{37}$$

$$qg \to q\mathcal{U},\ \overline{q}g \to \overline{q}\mathcal{U} \tag{38}$$

These are partonic subprocesses. To consider these sub-processes involving both the quark and gluon, we consider our vector unparticle operator $O_{\mathcal{U}}^{\mu}$. Meanwhile, for gluon-gluon sub-processes, we consider the scalar operator $O_{\mathcal{U}}$ [6].

The cross-section for the partonic process is

$$\frac{d^2\hat{\sigma}}{d\hat{t}dP_{\mathcal{U}}^2} = \frac{1}{16\pi\hat{s}^2}|\bar{\mathcal{M}}|^2\frac{1}{2\pi}A_{d_{\mathcal{U}}}\left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2}\frac{1}{\Lambda_{\mathcal{U}}^2} \tag{39}$$

with differing $|\bar{\mathcal{M}}|^2$ for different processes [7]

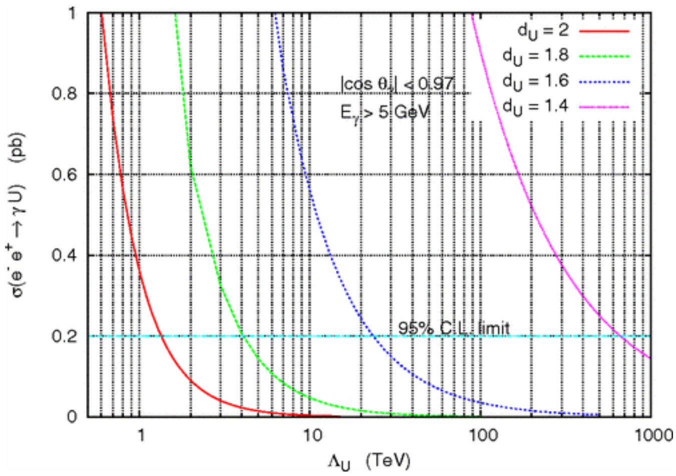

Figure 2: Cross sections for mono-photon plus unparticle production at the e^+e^- collider with $\sqrt{s} = 207$ GeV for $d_\mathcal{U} = 1.4, 1, 6, 1.8$ and 2. The horizontal line of 0.2 pb is the 95% C.L. upper limit, reproduced from [7].

$$
\begin{aligned}
|\bar{\mathcal{M}}|^2(gg \to g\mathcal{U}) &= \frac{1536\pi\alpha_s}{4\cdot 8\cdot 8}\lambda_o^2 (P_\mathcal{U}^2)^4 + \hat{s}^4 + \hat{t}^4 + \hat{u}^4, \\
|\bar{\mathcal{M}}|^2(q\bar{q} \to g\mathcal{U}) &= \frac{8}{9} g_s^2 \lambda_1^2 \frac{(\hat{t} - P_\mathcal{U})^2 + (\hat{u} - P_\mathcal{U})^2}{\hat{t}\hat{u}}, \\
|\bar{\mathcal{M}}|^2(qg \to q\mathcal{U}) &= -\frac{1}{3} g_s^2 \lambda_1^2 \frac{(\hat{t} - P_\mathcal{U})^2 + (\hat{s} - P_\mathcal{U})^2}{\hat{t}\hat{s}}.
\end{aligned} \tag{40}
$$

These matrix elements, squared, for process written to right also provide a similar form for $\bar{q}g \to \bar{q}\mathcal{U}$. Meanwhile, the first process, which is $gg \to g\mathcal{U}$ with λ_o^2, will have extra-dimensional regularization and suppression because of dimensional counting. In these partonic subprocesses, we say $\hat{s} \sim x_1 x_2 s$ with s being the center of mass squared of hadrons colliding and $x_1 x_2$ is momentum parton function. For $\Lambda_\mathcal{U} = 1TeV$ and $\lambda_0\lambda_1 = 1$, there is an analysis of this mono-jet at LHC. However, the phase factor $A_{d_\mathcal{U}}$ seems to not function at the partonic hadron colliding process. Furthermore, this mono-jet also does not tell us more about hadronic contribution for unparticle detection, so we cannot be sure to find unparticle, one of the cause is unknown $\hat{s}$ in these equations, at LHC[5].

3.5 Constraints on $A_{d_\mathcal{U}}$ for mono-photon production

Here, we will study about the constraints by LEP by inserting unparticle to their observed missing energy. At LEP2 [10–13], the Standard Model predicts that events with one (or more) photons with invisible particles can be found by the process of $e^+e^- \to X + \gamma$. L3 found with 95% C.L

[5]For spin-2 unparticle, the theory should be same and hence should not alter our perception, at least here.

that upper bound on $\sigma(e^+e^- \to X + \gamma) \simeq 0.2$pb under the cut of $E_\gamma > 5$GeV and $cos\theta_\gamma < 0.97$ at $\sqrt{s} = 207$ GeV and we will work on it. [7] presents a study of mono-photon with unparticle with the same cuts defined with $\sqrt{s} = 207$ GeV versus the unparticle scale $\Lambda_\mathcal{U}$ with defined $\lambda_1 = 1$, which is Fig 2.

A similar table can also be put down for $\Lambda_\mathcal{U}$ for different $d_\mathcal{U}$ for LEP2 with 95% C.L.

$d_\mathcal{U}$	$\Lambda_\mathcal{U}$ (TeV scale)
2.0	1.35
1.8	4
1.6	23
1.4	660

A simple observation yields to us that for fractional numbers of unparticles, we need higher energies. We will not discuss more the phenomenological processes in the unparticle interaction, and it has been studied in detail [7]. And, the process $e^+e^- \to \mu^+\mu^-$ in [2].

4 Conclusion

What we have so far discussed are theoretical cross-sections of our probable processes. In the first section, we introduced "Unparticle Physics" as a field theory with scale invariance, which arises from a weakly coupled Bank-Zaks field. Unparticle operators are studied. We find out that unparticles can exist in non-integral numbers and that what sets the unparticle to be different from any normal field theory. Then, we studied other standard model processes and found them either suppressed or at high energies.

We also see that unparticles can play an essential role in standard model processes, as they can act as missing energies in our standard model anomalies. We hope to find them in our colliders someday. And if we do that, it will be another milestone for particle physics. A lot is going on [14, 15] in unparticle physics that hints at it to be an exceptional field theory. In particular, recently introduced term "Unnuclear Physics" [14] is another promise of *non-relativistic* unparticle physics.

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