Note; Singlet means that it is invariant under SU(N) group.

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→ Heterotic theory.

Gross, Harvey, Martinec and Rohm.

We decouple left and right-moving modes. Where we start with a superstring theory, where Right moving modes are same like previous modes discussed.

But left-moving modes to include a suitable current algebra. This will introduce non abelian gauge symmetries. This hybridization is called "heterosis".

We describe an action related to this

$$S = -\frac{1}{2\pi}\int d^2\sigma \left(\sum_{\mu=0}^{9} \left(\partial_\alpha X^\mu \partial^\alpha X_\mu - 2i\psi^\mu_- \partial_+ \psi_{\mu-} \right) \underbrace{- 2i \sum_{A=1}^{n} \lambda^A_+ \partial_- \lambda^A_+}_{\text{left-moving modes}} \right).$$

ψ^μ, $\mu = 0, \ldots, 9$ transform as the vector rep. of the Lorentz group. λ^A $A = 1, \ldots, n$ are Lorentz singlets, which carry some internal charges.

Both λ^A and ψ^μ are Majorana-Weyl fermions.

The right-moving modes are ψ_-^μ and the right-moving part of X^μ. These are the right-moving modes of one of the type II models, so the critical dimension is ten, this is why we have set $D=10$ in the action.

There is also a supersymmetry between X^μ and ψ_-^μ.

$$\delta X^\mu = i\epsilon\psi_-^\mu \qquad \delta\psi_-^\mu = \epsilon\partial_- X^\mu$$

ϵ has only positive chirality here. This introduces the commuting ghosts. Thee modes are $v_{3/2}$ and $u_{-1/2}$.

The left moving modes are from X^μ and λ^A. However, there are no supersymmetries in left-moving sector. The only left-moving ghosts are reparametrization ghosts, which are enough to cancel the contribution of 26 bosons.

But since, we have only ten X^μ, the rest of the Virasoro anomaly must be cancelled by λ^A. 32 λ^A is needed. With same boundary conditions, λ^A carry SO(32) gauge theory | If the boundary conditions are different, one instead gets $E_8 \times E_8$.

& some boundary conditions!

SO(32) Theory

There will be two conditions; periodic and anti-periodic. One ~~may~~ can break SO(32) with assigning periodic boundary conditions to some left-moving modes and some anti-periodic.

→ periodic sector, P, is analog of Ramond sector

$$\lambda^A(\sigma) = \sum_{-\infty}^{\infty} \lambda_n^A e^{-2in\sigma}$$

$n \in Z$ and

$$\{\lambda_m^A, \lambda_n^B\} = \delta^{AB}\delta_{m+n}$$

→ The anti-periodic sector, A, is analog of NS sector (Superstring) → modes λ_r^A with $r \in Z + 1/2$ and the anti-commutation relations

$$\{\lambda_r^A, \lambda_s^B\} = \delta^{AB}\delta_{r+s}$$

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We can construct separate Virasoro operators L_m and $\tilde{L}_m$ from right and left-moving modes.

A physical state $|\Omega\rangle$ was required to obey $[L_m|\Omega\rangle = \tilde{L}_m|\Omega\rangle = 0]$ for $m>0$

$$\boxed{(L_0 - a)|\Omega\rangle = (\tilde{L}_0 - \tilde{a})|\Omega\rangle = 0}$$

L_0 (or $\tilde{L}_0$) is of form $p^2/8 + N$ (or $p^2/8 + \tilde{N}$) where N and $\tilde{N}$ are constructed from oscillators coordinates. In terms of transverse modes

$$N = \sum_1^\infty (\alpha_{-n} \cdot \alpha_n + n S^a_{-n} S^a_n)$$

for left-moving modes on the other hand (for p-sector)

$$\tilde{N} = \sum_1^\infty (\tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n + n \lambda^A_{-n} \lambda^A_n)$$

and for (A sector)

$$\tilde{N} = \sum_1^\infty (\tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n + \lambda^A_{-r} \lambda^A_r)$$

For constants a and $\tilde{a}$, we have $\boxed{a=0}$ from supersymmetry.

$$\boxed{\begin{aligned} \tilde{a}_A &= \frac{8}{24} + \frac{32}{48} = 1 \\ \tilde{a}_P &= \frac{8}{24} - \frac{32}{24} = -1 \end{aligned}}$$

Since $a=0$, we have a on-shell mass condition

$$p^2 = -8N \qquad \text{— a}$$

No tachyons

~~On~~ In A-Sector

$$\frac{1}{4}(\text{mass})^2 = N + \tilde{N} - 1$$

where $N = \tilde{N} - 1$, and in the P sector

$$\frac{1}{4}(\text{mass})^2 = N + \tilde{N} + 1$$

where $N = \tilde{N} + 1$.

According to (a), we have massless states for $N=0$, So we must have massless products in the A sector with $\tilde{N} = 1$, while in P sector with $\tilde{N} = -1$.

(But since $\tilde{N}$ is a semi-definite operator, we ignore P-sector).

The space of massless sector is just the tensor products of right-moving modes with $N=0$ and with the left-moving modes with $\tilde{N} = 1$ (in the A sector).

P sector
is as important
as A sector, will
see in loop diagrams.

$Spin(32)/Z_2$

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For the right-moving modes, the space of states $N=0$ with $D=10$ susy yang-mills multiplet.

$$|i\rangle_R \text{ and } |a\rangle_R$$

$$\uparrow \qquad\qquad \uparrow$$

$$8_V \qquad\qquad 8_C$$

of Spin(8)

For the left-moving modes with $\tilde{N}=1$ (massless)

$$\alpha^i_{-1}|0\rangle_L \longrightarrow 8 \text{ transverse states}$$

Spin(32)

These states are, of course, SO(32) singlets of course.
And, other state for $\tilde{N}=1$

$$\lambda^A_{-1/2}\lambda^B_{-1/2}|0\rangle_L \text{ — } 16 \text{ states.}$$

These are Lorentz singlets and transform in the adjoint representation of SO(32)

Lie ~~algebras~~ groups → a manifold differential
i.e. symmetries which are continuous

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$E_8 \times E_8$ Theory

Finite dimensional Lie algebra. Most exceptional as they say.

E_8 ~ Exceptional, E_8 is the largest (of course A_{1000} is).

We begin with $SO(16)$ sub-algebra. The generators of $SO(16)$ are operators J_{ij} (with $J_{ij} = -J_{ji}$ so that there are $\frac{16 \times 15}{2} = 120$ of them) that obey the $SO(16)$ Lie algebra

$$[J_{ij}, J_{kl}] = J_{il}\delta_{jk} - J_{jl}\delta_{ik} - J_{ik}\delta_{jl} + J_{jk}\delta_{il}.$$

To these we adjoin operators Q_α transforming in the positive chirality spinor rep. of $SO(16)$.

$SO(16)$ dim is $2^7 = 128$

$$\text{so } \boxed{128 + 120 = 248}$$

Q_α transforms as spinors of $SO(16)$ means that

$$\boxed{[J_{ij}, Q_\alpha] = (\sigma_{ij})_{\alpha\beta} Q_\beta}$$

Spin(n) 2,3!

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We must define a Lie-algebra, so not super-algebra, So this is a commutator, not an anti-commutator!

$SO(16)$ group theory uniquely determines this is up to normalization to be.

$$[Q_\alpha, Q_\beta] = (\sigma_{ij})_{\alpha\beta} J_{ij}$$

In constructing the spin $(32)/Z_2$ theory we assigned to all 32 components of λ^A the same boundary conditions — Aoop — in order to be consistent with the symmetry.

$E_8 \times E_8$ has 496 generators, as $SO(32)$.
$E_8 \times E_8$ is rank "8".

[$E_8 \times E_8$ is far more interesting than $SO(32)$.]

⇒ We construct a theory with smaller symmetry elements than $Spin(32)$, Perhaps $Spin(n) \times Spin(32-n)$. We remove the 32 left-moving fermions up into a group of n and a group of $32-n$.

So

$SO(n) \times SO(32-n)$ doesn't have same boundary conditions (P and A sector).

$E_8 \times E_8$ Theory — Continue

So, we can assign the P sector and A sector boundary conditions to the two sets of fermi oscillators separately → for $SO(n) \times SO(32-n)$

4 possible sector
i) AA
ii) AP
iii) PA
iv) PP

where the first label refers to the boundary conditions of $SO(n)$ of "λ" and the second label to those obeyed by the remaining $32-n$ components.

four relations for
$N = \tilde{N} - \tilde{a}$ as well.

$$\tilde{a}_{AA} = 1$$

$$\tilde{a}_{AP} = \frac{n}{16} - 1$$

$$\tilde{a}_{PA} = 1 - \frac{n}{16}$$

$$\tilde{a}_{PP} = 1$$

See the EWS

So far, spin(16) × spin(16) is the algebra for $E_8 \times E_8$.

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For AA sector and PP sector, we have

$n = 8$ or $24 \rightarrow Spin(24) \times Spin(8)$

$n = 16 \longrightarrow Spin(16) \times Spin(16)$

$n = 32 = 0 \rightarrow Spin(32)/Z_2$.

o We will ignore spin(24) × spin(8) as this suffers a lot from one-loop amplitudes and irregularities.

We have two oscillators (each contributing +1/2 to the eigenvalue of $\tilde{N}$)

$$\boxed{\lambda^A_{-1/2}\, \lambda^B_{-1/2}\, |0\rangle_L}$$

The spectrum.

Under Spin(16) × Spin(16)

$(120, 1)$ if $A, B = 1, \ldots, 16$

$(1, 120)$ if $A, B = 17, \ldots, 32$.

$(16, 16)$ if $A = 1, \ldots, 16$, $B = 17, \ldots, 32$.